prediction of Dellinger is slightly higher than the present shock-slip result using common reaction rates. Based on Fig. 3, it becomes obvious that the large difference between Lee and Zierten[1] and Dellinger[3] cannot be attributed solely to differences in reaction rates. Hence, the method of solution (coupled vs decoupled) must be important as suggested by Dellinger[3] in his paper.

Conclusions

Nonequilibrium, multicomponent air calculations for electron-density distributions in the stagnation point, merged viscous shock layer have been presented using a shock-slip analysis in conjunction with a coupled, iterative finite-difference numerical solution. The present study yielded four main conclusions summarized as follows: 1) Including of shock-slip effects are necessary for analysis of merged viscous-layer electron density using a thin viscous shock-layer model. Such a shock-slip approach appears to be valid based on comparison with solutions which include details of the shock structure. 2) Predicted electron-density distributions are relatively insensitive to choice of diffusion model (constant Lewis-Semenov number for all species vs full multicomponent diffusion). 3) For an equilibrium catalytic-wall condition, a factor of four maximum difference in peak electron density resulted from the four different sets of reaction rates used in the present study. The catalytic condition of the wall influences the entire shock layer with the principal effect concentrated in the near-wall region. 4) Decoupled calculations in which the species concentrations are solved for using frozen-flow fluid-dynamic solutions appear to overpredict the electron density as compared to coupled calculations such as that of Dellinger[3] and the present work.

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Acoustic Tracking of Supersonic Objects

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Introduction

In this Note equations are developed for determining the location, speed, and direction of motion of an object traveling faster than the speed of sound from the times at which the shock wave arrives at some microphones.

Solutions to this problem have been given previously by Zaroodny[1] and Reid.[2] However, the method of Ref. 2 was found to be impractical to use because it was too sensitive to small errors in the locations of the microphones and/or in the recorded times at which the shock wave arrived at the microphones. The purpose of the present Note is to give a calculation procedure in which acceptable accuracy is more easily obtained. In Ref. 2, all the details about the missile path were determined from a single cluster of microphones. In this Note, less information is obtained from a cluster of microphones, and three or more of them are used. They may be widely separated. This in itself results in a significant improvement in the accuracy of tracking when there are small errors in the data. In addition, the present mathematical analysis permits extra readings to be used easily. With exactly three clusters of microphones, Eqs. (2) and (4) below, each give three equations in three unknowns and serve to determine the missile's velocity and location. If more than three clusters of microphones are used, more than three equations will be obtained from Eqs. (2) and (4), and the method of least squares can be used to obtain best values for the calculated velocity and location.

Analysis

Assume that an object is traveling in a straight line in a fluid with a constant speed v greater than the speed of sound in the medium, and that it generates a shock wave that is a right circular cone except perhaps in the neighborhood of the object. Let it be desired to find the speed and line of flight of the vertex of the cone, and hence of the object, from the times t at which the shock wave arrives at various microphones.

Let three lines of microphones be placed so as to pass through a point with the lines parallel respectively to the x, y, and z axes. This cluster of microphones will be a listening station. Use the microphones to find the time at which the shock cone arrives at the intersection of the rows of microphones, and also the partial derivatives of t with respect to x, y, and z at that point, and therefore ∇t.

∇t is a vector perpendicular to the surface t = const, and hence to the shock cone. Since a shock wave moves in the direction of ∇t with the speed of sound c it follows that $|\nabla t| = 1/c$, or

$$(\partial t/\partial x)^2 + (\partial t/\partial y)^2 + (\partial t/\partial z)^2 = 1/c^2 \tag{1}$$

Thus, the partial derivatives of t at a point are dependent. One of them may be calculated from the other two, except for sign, so that three lines of microphones are not needed at a listening station, but only two.

The object moves a distance vT along the axis of the cone in time T while the shock wave is traveling a distance cT perpendicular to itself, and hence perpendicular to an element of the cone. Thus, the cosine of the angle between $\mathbf{v}$ and ∇t is c/v. Therefore

$$\mathbf{v}\cdot\nabla t = 1 \tag{2}$$

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Determine ∇t at three different points, and assume that these vectors are independent. The three components of $\mathbf{v}$ can then be calculated by applying Eq. (2) at each of the listening stations. Note that the magnitude and direction of the speed of the object are thus obtained from Eq. (2) without using the locations of the listening stations.

Let $\mathbf{R}$ be a vector from the origin of coordinates to the vertex of the cone at time $t = 0$, and $\mathbf{L}$ a vector to a listening station. Then

$$\mathbf{L} = \mathbf{R} + \mathbf{v}t + \mathbf{S} \tag{3}$$

where $\mathbf{S}$ is an element of the cone extending from its vertex at time t to the listening station lying on the cone at that time. Thus,

$$\mathbf{R}\cdot\nabla t = \mathbf{L}\cdot\nabla t - t \tag{4}$$

The components of $\mathbf{R}$ may be calculated by solving simultaneously the three equations obtained by using Eq. (4) at each listening station.

This completes the determination of $\mathbf{v}$ and $\mathbf{R}$, and hence solves the problem of locating the object at any time. If in addition an explicit expression is desired for $\mathbf{v}$ or $\mathbf{R}$, it may be obtained in terms of the observed gradients by using the three cross products of them taken two at a time as a basis. By using Eq. (2), the velocity $\mathbf{v}$ is easily found in this way to be given by

$$\mathbf{v}[(\nabla t)_1\cdot(\nabla t)_2 \times (\nabla t)_3] = (\nabla t)_1 \times (\nabla t)_2 + (\nabla t)_2 \times (\nabla t)_3 + (\nabla t)_3 \times (\nabla t)_1 \tag{5}$$

A similar equation for $\mathbf{R}$ can be derived by applying Eq. (4) to get the coefficients of the cross products.

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Base Heat Transfer in an Axisymmetric Supersonic Flow

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THERE have been a number of theoretical and experimental studies of heat transfer in two-dimensional supersonic separated flows. Although some of these two-dimensional theoretical studies have been extended to the axisymmetric case,[1–3] few axisymmetric data are available for comparison. This Note presents the results of an experimental investigation of blunt-base heat transfer in an axisymmetric turbulent supersonic separated flow, and compares the results with published data and with recently published correlating parameters for both base heating and base cooling.

The experimental investigation was conducted in the Rutgers Axisymmetric Near-wake Tunnel (RANT), which is shown in Fig. 1. This vertical blow-down facility was designed specifically for the investigation of turbulent, supersonic near wakes of axisymmetric models in the absence of the usual support interference problems. The annular nozzle, incorporating a 3-in.-diam. cylindrical centerbody, was designed by the method of characteristics to produce a uniform flowfield at Mach 4.0. Models were attached to an appropriate centerbody extension, thus locating the model base at a position of optimum flow conditions. Detailed discussions of the construction and operation of the facility have been previously reported by Przirembel,[4] and Sieling, Przirembel, and Page.[5]

Steady-state stagnation pressure for RANT was maintained constant within $\pm\frac{1}{2}$ psia during each run, with run times of approximately 20 sec. Stagnation temperatures decreased at a rate of approximately 1°R/sec during each run. All tests in this investigation were conducted at a stagnation pressure of 152.2 psia and an average stagnation temperature of 510°R $\pm$ 10°R. The resulting freestream Reynolds number, 1.6×10^7 per ft, indicates a completely turbulent approaching boundary layer.

Three base-heat-transfer models were fabricated by using a steel collar and an appropriately machined Bakelite insert, as shown in Fig. 2. Thin copper heat meters, instrumented with 30 AWG copper-constantan thermocouples, were glued to the Bakelite insert. The blunt base of each model assembly was then polished to a uniform surface.

The average heat-transfer coefficient on each heat-meter surface was determined by a transient technique, utilizing an energy balance of the heat meter and assuming that the energy transfer was a quasi-steady process. An expression for the convective heat transfer then was derived in terms of the temperature-time gradient, the base temperature, and the tunnel stagnation temperature. The average heat-transfer coefficient for each heat meter was then obtained by measuring the instantaneous temperature and the rate of temperature change. A more detailed description of the model fabrication and experimental techniques employed has been reported previously.[6]

The average heat-transfer rates for a blunt base immersed in an axisymmetric flow were obtained with model HT 1. The heat-transfer rates measured with model HT 2 resulted in local heat-transfer coefficients situated at the coordinates of each heat meter, as shown in Fig. 2. The average heat-transfer coefficient for each heat meter then was considered a local heat-transfer coefficient located at a fixed point in the base region, and integrated over the base area to find the average heat-transfer coefficient for the base. Variation in heat-transfer coefficient due to radial change, or radial location, was measured with model HT 3.

Similar experimental heat-transfer coefficients were obtained with heat meters located 180° apart on model HT 2. This result, indicating a uniform angular heat-transfer rate, was a check on the axisymmetric flow conditions. Integration of these local values over the area of the base resulted in average heat-transfer values consistent with the results obtained with model HT 1.

The experimental heat-transfer coefficients obtained with model HT 3 indicated that the peak heating rate occurred

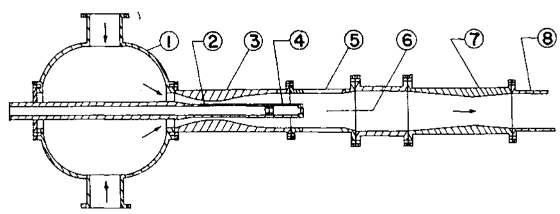

Fig. 1 Schematic of the Rutgers Axisymmetric Near-wake Tunnel (RANT): 1) settling chamber, 2) upstream sting, 3) nozzle, 4) model, 5) windows, 6) test section, 7) diffuser, 8) exhaust.

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